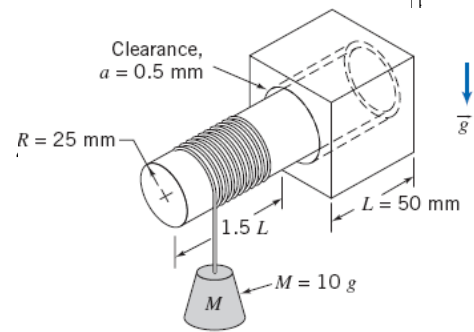


# Problem 2.65

[Difficulty: 4] Part 1/2

**2.65** A circular aluminum shaft mounted in a journal is shown. The symmetric clearance gap between the shaft and journal is filled with SAE 10W-30 oil at  $T = 30^\circ\text{C}$ . The shaft is caused to turn by the attached mass and cord. Develop and solve a differential equation for the angular speed of the shaft as a function of time. Calculate the maximum angular speed of the shaft and the time required to reach 95 percent of this speed.



Solution: Apply summation of torques and Newton's second law.

Basic equations:  $\Sigma T = I \frac{d\omega}{dt}$        $\Sigma F = m \frac{dv}{dt}$        $v = R\omega$

For the mass:

$$\Sigma F_y = mg - t = m \frac{dv}{dt} = mR \frac{d\omega}{dt} \quad (1)$$

For the shaft:

$$\Sigma T = tR - T_{\text{viscous}} = I \frac{d\omega}{dt} \quad (2)$$

$$T_{\text{viscous}} = \tau A = \mu \frac{v}{a} R 2\pi RL = \frac{2\pi\mu\omega R^3 L}{a}$$

Assume: (1) Newtonian liquid, (2) small gap, (3) linear profile

Then Eq. 2 becomes  $tR - \frac{2\pi\mu R^3 L}{a} \omega = I \frac{d\omega}{dt}$  ;  $I = \frac{1}{2} MR^2$  (3)

Multiplying Eq. 1 by R and combining with Eq. 3 gives

$$mgR - mR^2 \frac{d\omega}{dt} - \frac{2\pi\mu R^3 L}{a} \omega = I \frac{d\omega}{dt} \quad \text{or} \quad mgR - \frac{2\pi\mu R^3 L}{a} \omega = (I + mR^2) \frac{d\omega}{dt} \quad (4)$$

This may be written  $A - B\omega = C \frac{d\omega}{dt}$  where  $A = mgR$ ,  $B = \frac{2\pi\mu R^3 L}{a}$ ,  $C = I + mR^2$

Separating variables  $\frac{d\omega}{A - B\omega} = \frac{dt}{C}$

Integrating  $\int_0^\omega \frac{d\omega}{A - B\omega} = -\frac{1}{B} \ln(A - B\omega) \Big|_0^\omega = -\frac{1}{B} \ln(1 - \frac{B\omega}{A}) = \int_0^t \frac{dt}{C} = \frac{t}{C}$

Simplifying  $1 - \frac{B\omega}{A} = e^{-Bt/C}$  or  $\omega = \frac{A}{B} [1 - e^{-Bt/C}]$  (5)  $\omega(t)$

The maximum angular speed ( $t \rightarrow \infty$ ) is  $\omega = A/B$ .

$$A = mgR = 0.010 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2} \times 0.025 \text{ m} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} = 2.45 \times 10^{-3} \text{ N} \cdot \text{m}$$

$$B = \frac{2\pi\mu R^3 L}{a} = 2\pi \times 0.095 \frac{\text{kg}}{\text{m} \cdot \text{s}} \times (0.025 \text{ m})^3 \times 0.050 \text{ m} \times \frac{1}{0.0005 \text{ m}} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} = 9.33 \times 10^{-4} \text{ N} \cdot \text{m} \cdot \text{s}$$

# Problem 2.65

[Difficulty: 4] Part 2/2

$$\text{Evaluating, } \omega_{\max} = \frac{A}{B} = 2.45 \times 10^{-5} \text{ N}\cdot\text{m} \times \frac{1}{9.33 \times 10^{-4} \text{ N}\cdot\text{m}\cdot\text{sec}} = 2.63 \text{ rad/s}$$

Thus

$$\omega_{\max} = 2.63 \frac{\text{rad}}{\text{s}} \times \frac{\text{rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{\text{min}} = 25.1 \text{ rpm}$$

$\omega_{\max}$

$$\text{From Eq. 5, } \omega = 0.95 \omega_{\max} \text{ when } e^{-Bt/C} = 0.05, \text{ or } Bt/C \approx 3; t \approx \frac{3C}{B}$$

$$C = I + mR^2 = \frac{1}{2}MR^2 + mR^2 = (\frac{1}{2}M + m)R^2$$

$$M = \pi R^2(1.5L + L)\rho = 2.5\pi R^2 L \rho$$

$$M = 2.5\pi \times (0.025)^2 \text{ m}^2 \times 0.050 \text{ m} \times (2.64)1000 \frac{\text{kg}}{\text{m}^3} = 0.648 \text{ kg}$$

$$C = (\frac{1}{2} \times 0.648 \text{ kg} + 0.010 \text{ kg})(0.025)^2 \text{ m}^2 = 2.09 \times 10^{-4} \text{ kg}\cdot\text{m}^2$$

Thus

$$t_{95} = 3 \times 2.09 \times 10^{-4} \text{ kg}\cdot\text{m}^2 \times \frac{1}{9.33 \times 10^{-4} \text{ N}\cdot\text{m}\cdot\text{s}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 0.671 \text{ s}$$

$t_{95}$

{ The terminal speed could have been computed from Eq. 4 by }  
 { setting  $d\omega/dt \rightarrow 0$ , without solving the differential equation. }